Space and Time: Einstein and Beyond

Every civilization has been fascinated by notions of Space (the Heavens) and Time (the Beginning, the Change, and the End).

Early thinkers from Lao Tsu to Aristotle wrote extensively on the subject. Over centuries, the essence of these commentaries crystallized in human consciousness, providing us with mental images that we commonly use. We think of space as a three-dimensional continuum which envelops us. We think of time as flowing serenely, all by itself, unaffected by forces in the physical universe. Together, they provide a stage on which the drama of interactions unfolds. The actors are everything else—stars and planets, radiation and matter, you and me.

These notions reigned for over two thousand five hundred years. In the middle of the 19th century, however, mathematicians discovered that Euclid’s geometry, which we all learned in school, is only one of many possible geometries. This discovery led to the idea, expounded most eloquently by Bernhard Riemann in 1854, that the geometry of physical space may not obey Euclid’s axioms—it may be curved due to the presence of matter in the universe. It took another 61 years for this idea to be realized in detail.

The grand event was Einstein’s publication of his general theory of relativity in 1915. In this theory, space and time fuse to form a four-dimensional continuum. The geometry of this continuum is curved and the amount of curvature in a region encodes the strength of the gravitational field there. Spacetime is not an inert entity. It acts on matter and can be acted upon. As the American physicist John Wheeler puts it: “Matter tells spacetime how to bend and spacetime tells matter how to move.” There are no longer any spectators in the cosmic dance, nor a backdrop on which things happen. The stage itself joins the troupe of actors. This is a profound paradigm shift. Since all physical systems reside in space and time, this shift shook the very foundations of natural philosophy. It has taken decades for physicists to come to grips with the numerous ramifications of this

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Gravity Is Geometry

Einstein was motivated by two seemingly simple observations. First, as Galileo demonstrated through his famous experiments at the leaning tower of Pisa, the effect of gravity is universal: all bodies fall the same way if the only force on them is gravitational. Second, gravity is always attractive. This is in striking contrast with, say, the electric force where unlike charges attract while like charges repel. As a result, while one can easily create regions in which the electric field vanishes, one cannot build gravity shields. Thus, gravity is omnipresent and nondiscriminating; it is everywhere and acts on everything the same way. These two facts make gravity unlike any other fundamental force and suggest that gravity is a manifestation of something deeper and universal. Since spacetime is also omnipresent and the same for all physical systems, Einstein was led to regard gravity not as a force but a manifestation of spacetime geometry.

Space-time of General Relativity is supple and can be visualized as a rubber sheet, bent by massive bodies. The Sun, for example, being heavy, bends space-time enormously. Planets like Earth move in this curved geometry. In a precise mathematical sense, they follow the simplest trajectories called geodesics—generalizations of straight lines of the flat geometry of Euclid to the curved geometry of Riemann. So, when viewed from the curved spacetime perspective, Earth takes the straightest possible path. But since spacetime itself is curved, the trajectory appears elliptical from the flat-space perspective of Euclid and Newton.

The magic of General Relativity is that, through elegant mathematics, it transforms these conceptually simple ideas into concrete equations and uses them to make astonishing predictions about the nature of physical reality. It predicts that clocks should tick faster in Denver than in State College. Galactic nuclei should act as giant gravitational lenses and provide spectacular, multiple images of distant quasars. Two neutron stars orbiting around each other must lose energy through ripples in the curvature of spacetime caused by their motion and
spiral inward in an ever tightening embrace. Over the last thirty years, astute measurements have been performed to test if these and other even more exotic predictions are correct. Each time, General Relativity has triumphed. The accuracy of some of these observations exceeds that of the legendary tests of quantum electrodynamics. This combination of conceptual depth, mathematical elegance, and observational successes is unprecedented. This is why General Relativity is widely regarded as the most sublime of all scientific creations.

**Big Bang and Black Holes**

General Relativity ushered in the era of modern cosmology. At very large scales, the universe around us appears to be spatially homogeneous and isotropic. This is the grandest realization of the Copernican principle: our universe has no preferred place nor favored direction. Using Einstein’s equations, in 1922 the Russian mathematician Alexander Friedmann showed that such a universe cannot be static. It must expand or contract. In 1929 the American astronomer Edwin Hubble found that the universe is indeed expanding. This in turn implies that it must have had a beginning where the density of matter and curvature of spacetime were infinite. This is the Big Bang. Careful observations, particularly over the last decade, have shown that this event must have occurred some 14 billion years ago. Since then, galaxies are moving apart, the average matter content is becoming dilute.

By combining our knowledge of General Relativity with laboratory physics, we can make a number of detailed predictions. For instance, we can calculate the relative abundances of light chemical elements whose nuclei were created in the first three minutes after the Big Bang; we can predict the existence and properties of a primordial (the cosmic microwave background) that was emitted when the universe was some 400,000 years old; and we can deduce that the first galaxies formed when the universe was a billion years old. An astonishing range of scales and variety of phenomena! In addition, General Relativity also changed the philosophical paradigm to phrase questions about the Beginning.

It’s not just matter but the spacetime itself that is born at the Big Bang. In a precise sense, the Big Bang is a boundary, a frontier, where spacetime ends. General Relativity declares that physics stops there; it does not permit us to look beyond.

\[ \ell_{pl} = \sqrt{\frac{G \hbar}{c^5}} \approx 10^{-33} \text{ cm} \]

The apparent continuum of spacetime modeled by the equations of General Relativity breaks down at the so called Planck scale—the unique length that can be constructed from Newton’s constant of gravitation \( G \), Planck’s constant of quantum physics \( \hbar \), and the speed of light \( c \).

black holes. As late as 1939, he published a paper in the *Annals of Mathematics* arguing that black holes could not be formed by the gravitational collapse of a star. The calculation is correct but the conclusion is an artifact of a nonrealistic assumption. Just a few months later, American physicists Robert Oppenheimer and Hartland Snyder published their now classic paper establishing that black holes do, in fact, result. These are regions in which the spacetime curvature is so strong that even light cannot escape. Therefore, according to General Relativity, to outside observers they appear pitch black. In the rubber-sheet analogy, the bending of spacetime is so extreme inside a black hole that spacetime is torn apart, forming a singularity. As at the Big Bang, curvature becomes infinite. Spacetime develops a final boundary and the physics of General Relativity simply stops.

And yet, black holes appear to be mundanely common in the universe. General Relativity, combined with our knowledge of stellar evolution, predicts that there should be plenty of black holes with about 10 solar masses, the end products of lives of large stars. Indeed, black holes are prominent players in modern astronomy. They provide the powerful engines for the most energetic phenomena in the universe such as the celebrated gamma-ray bursts in which an explosion spews out, in a few blinding seconds, as much energy as a 1,000 Suns.

Abhay Ashtekar has been a member of the Penn State faculty since 1994. He is Holder of the Eberly Family Chair in Physics and director of the Penn State Institute for Gravitational Physics and Geometry. He is recognized for his contributions both to Einstein’s classical theory of gravitation, or General Relativity, and to the ongoing effort to create a quantum theory of gravity. He has contributed substantially to the interplay between geometry and physics, particularly in the analysis of black holes. He reformulated General Relativity using new variables, which have been named after him. They enable him and his collaborators to make major progress toward the development of a quantum theory of gravity. This work has led to a new mathematical description of the structure of spacetime as polymer-like at the smallest scale. Before coming to Penn State, he held positions at Syracuse University, the University of Paris in France, the University de Clermont-Ferrand in France, the University of Chicago, and the University of Oxford in the United Kingdom.
In this artist’s representation of quantum space-time, time runs vertically. General Relativity provides only the upper half of this space-time, which originates in a big-bang, depicted by the band in the middle. Quantum Einstein’s equations extend this space-time to the past of the big-bang, i.e., below the band.

Beyond Einstein

General Relativity is the best theory of gravitation and spacetime structure we have today. It can account for a truly impressive array of phenomena. But it is incomplete because it ignores the quantum effects that govern the subatomic world. Moreover, the two theories are dramatically different. The world of General Relativity has geometric precision, it is deterministic; the world of quantum physics is dictated by fundamental uncertainties, it is probabilistic. Physicists maintain a happy, carefree attitude, using General Relativity to describe the large-scale phenomena of astronomy and cosmology and quantum mechanics to account for properties of atoms and elementary particles. This is a viable strategy because the two worlds rarely meet. Nonetheless, from a conceptual standpoint, this is highly unsatisfactory. Everything in our experience as physicists tells us that there should be a grander, more complete theory from which General Relativity and quantum physics arise as special, limiting cases. This would be the quantum theory of gravity. It would take us beyond Einstein.

Beyond Einstein

At the Big Bang and black-hole singularities, the world of the very large and of the very small meet. Therefore, although they seem arcane notions at first, these singularities are our gates to go beyond General Relativity. Real physics cannot stop there. Rather, General Relativity fails. We need to dramatically revise, once again, our notions of space and time. We need a new syntax.

Over the last decade, researchers at the Institute of Gravitational Physics and Geometry at Penn State have played a lead role in creating such a syntax which, for historical reasons, goes under the name Loop Quantum Gravity. In General Relativity, spacetime is modeled by a continuum. The new idea is that this is only an approximation, which would break down at the so called Planck scale—the unique length that can be constructed from Newton’s constant of gravitation, Planck’s constant of quantum physics, and the speed of light.

This scale is truly minute, some 20 orders of magnitude smaller than the radius of a proton. Therefore, even in the highest-energy particle accelerators on Earth, one can safely work with a continuum. But the approximation would break down in more extreme situations; in particular, near the Big Bang and inside black holes. There, one must use a quantum spacetime of Loop Quantum Gravity.

What is a quantum spacetime? Look at the sheet of paper in front of you. For all practical purposes, it seems continuous. Yet we know that it is made of atoms. It has a discrete structure, which becomes manifest only if you zero in using, say, an electron microscope. Now, Einstein taught us that geometry is also a physical entity, on a par with matter. Therefore, it should also have an atomic structure. To unravel it, in the mid 90’s, researchers at the Penn State Institute of Gravitational Physics and Geometry and elsewhere combined the principles of General Relativity with quantum physics to develop a quantum geometry. It describes quantum spacetimes.

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mathematical theory in which the primary objects—the fundamental excitations of geometry—are one dimensional. Just as a piece of cloth appears to be a smooth, two-dimensional continuum although it obviously is woven by one-dimensional threads, the spacetime of General Relativity appears as a four-dimensional continuum although it is, in fact, a coherent superposition of these one-dimensional excitations. Intuitively, then, these fundamental excitations can be thought of as quantum threads that can be woven to create the fabric of spacetime. What happens, then, near spacetime singularities? There, the continuum approximation fails. The quantum fluctuations are so huge that quantum threads can no longer be frozen into a coherent superposition. The fabric of spacetime is ruptured. Continuum physics rooted in this fabric stops. But the quantum threads are still meaningful. Using a quantum generalization of Einstein’s equations, one can still do physics to describe what happens in the quantum world. In the absence of a spacetime continuum, many of the notions habitually used in physics are no longer available. New concepts have to be introduced, new physical intuition has to be honed. In this adventure, quantum equations pave the way.

Using this framework, over the past three years, the Big Bang has been analyzed in some detail both at the Penn State Institute of Gravitational Physics and Geometry and at the Albert Einstein Institute in Germany. It turns out that the partial differential equations of Einstein, adapted to the continuum, have to be replaced by difference equations, adapted to the discrete structures of quantum geometry. Except very near the Big Bang, equations of General Relativity provide an excellent approximation to the more fundamental ones. However, very near the Big Bang the approximation breaks down completely. In quantum geometry, spacetime curvature does become very large in this Planck regime, but not infinite. Gravity, in fact, becomes repulsive. General Relativity breaks down. But quantum Einstein’s equations enable us to “evolve” the quantum state of geometry and matter right through the singularity. In simple cases, where reliable numerical calculations have been performed, continuum is again a good approximation outside the Planck regime of the “other side of the Big Bang.” If this behavior turns out to be robust, the paradigm to pose questions about the Beginning will again be shifted. If the questions refer to the notion of time that Einstein gave us, there was indeed a Beginning. Not at the Big Bang though, but “a little later” when spacetime can be modeled as a continuum. But if by Beginning one means a firm boundary beyond which physical predictions are impossible, then the answer is very different from that given by General Relativity: in the more complete theory, there is no such Beginning.

To summarize, then, our understanding of space and time underwent a dramatic revision in the 20th century. But a new paradigm shift awaits us again in the 21st century. Thanks to quantum geometry, the Big Bang and black hole singularities are no longer final frontiers. The physical, quantum spacetime is much larger than what General Relativity had us believe. The existence of these new and potentially vast unforeseen domains already has provided a fresh avenue to resolve a 30-year-old problem at the foundation of physics, stemming from black-hole singularities. Even more exciting opportunities await us in the new questions and rich possibilities this extension opens up in cosmology.

Abhay Ashtekar

On discovery of General Relativity, the eminent mathematician and physicist Hermann Weyl wrote: “It is as if a wall which separated us from the truth has collapsed. Wider expanses and greater depths are now exposed to the searching eye of knowledge, regions of which we had not even a pre-sentiment.”

Penn State researchers in the Institute of Gravitational Physics and Geometry have played a lead role, world-wide, in extending the frontiers of General Relativity and in creating theories to take us beyond Einstein. With colleagues, post-docs, and students, Roger Penrose has made major contributions to our understanding of Quantum Gravity and black-hole theory; Peter Mészáros has made, and continues to make, seminal contributions to our understanding of gamma-ray bursts; Lee Samuel Finn, who is the director of Penn State’s Center for Gravitational Wave Physics, and Benjamin Owen are laying foundations of the new and exciting field of gravitational-wave astronomy; and Pablo Laguna, Deirdre Shoemaker, and Jinghao Xu use state-of-the-art computational methods to explore the fully nonlinear, strong-field gravity of General Relativity. Loop Quantum Gravity, pioneered at the Institute, is widely regarded as one of the most successful avenues to new physics, beyond General Relativity.