QUANTUM NATURE OF THE BIG BANG IN LOOP QUANTUM COSMOLOGY *

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1 Introduction

A central feature of general relativity is that gravity is encoded in the very geometry of space-time. Loop quantum gravity is a non-perturbative approach to unifying general relativity with quantum physics while retaining this interplay between geometry and gravity [1, 2, 3]. There is no background space-time; matter as well as geometry are 'quantum from birth'. Effects of quantum geometry are negligible under ordinary circumstances but they dominate near singularities. There, quantum space-time is dramatically different from the smooth continuum of general relativity. In particular, quantum geometry effects have led to a natural resolution of space-like singularities in a number of mini and midi-superspaces. These encompass both black hole and cosmological contexts.

In the cosmological setting, there are several long-standing questions that have been relegated to quantum gravity. Examples are:

• How close to the Big Bang does a smooth space-time of general relativity make sense? In particular, can one show from first principles that this approximation is valid at the onset of inflation?
• Is the Big-Bang singularity naturally resolved by quantum gravity? Or, is some external input such as a new principle or a boundary condition at the Big Bang essential?
• Is the quantum evolution across the 'singularity' deterministic? One needs a fully non-perturbative framework to answer this question in the affirmative. (In the Ekpyrotic and Pre-Big-Bang scenarios, for example, the answer is in the negative.)

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• If the singularity is resolved, what is on the 'other side'? Is there just a *quantum foam* far removed from any classical space-time, or, is there another large, classical universe?

Key results obtained by Bojowald over the past five years have shown that the big-bang singularity is indeed resolved by the quantum geometry effects that underlie loop quantum gravity [4, 5]. Recently, these results were extended significantly in models with a massless scalar field by first showing that the scalar field can serve as 'emergent time' and then rigorously constructing the physical Hilbert space, a complete set of (Dirac) observables and states which are semi-classical at late times [6]. This analysis has provided detailed answers to the long standing questions listed above.

2 Novel features of loop quantum cosmology

Quantum cosmology is an old subject. It was studied extensively in the framework of geometrodynamics where quantum states are taken to be functions of 3-geometries and matter fields. In the cosmological context, the wave functions $\Psi(a, \phi)$ depend on the scale factor a and the matter field ϕ. They are subject to a quantum constraint, called the Wheeler-DeWitt equation. Recall that in the case of the hydrogen atom, thanks to a non-zero $\hbar$, quantum effects tame the singular behavior of the classical ground state energy and render it finite, $E_o = -m^2e^4/2\hbar^2$. The hope was that a non-zero Planck-length would similarly tame classical curvature singularities through the Wheeler-DeWitt equation. Unfortunately, this hope was not realized. For example in the simplest of homogeneous isotropic models, if one begins with a semi-classical state at late times and evolves it back via this equation, one finds that it just follows the classical trajectory into the Big Bang singularity. These models have only a finite number of degrees of freedom whence, under standard assumptions, von Neumann's uniqueness theorem assures us that, up to unitary equivalence, the resulting quantum mechanics is unique. Therefore it was generally believed since the seventies that quantum cosmology can not lead to a natural resolution of the Big Bang singularity.

Loop quantum gravity is based on spin-connections rather than metrics and is thus closer in spirit to gauge theories. The basic dynamical variables are holonomies h of a gravitational spin-connection A, and electric fields E canonically conjugate to these connections. However, since the 'internal' (or gauge) index refers to spin, it is now tied to space-time. Consequently, the E's now have a dual, geometrical interpretation: they represent orthonormal triads which determine the Riemannian geometry. Thanks to contributions of 2 dozen or so groups since the mid-nineties, the subject has reached a high degree of mathematical precision [1]. In particular, it has been shown that the fundamental quantum algebra based on h's and E's admits a *unique* diffeomorphism covariant representation [7]. From the perspective of Minkowskian field theories, this result is surprising and brings out the powerful role of the requirement of diffeomorphism covariance (i.e., background independence). In this representation, quantum states depend on gravitational spin-connections and matter fields, but the dependence on connections is only through holonomies h. Hence there are well-defined holonomy operators $\hat{h}$ but it turns out that *there is no operator* $\hat{A}$ *corresponding to the connection itself.* The second key feature is that Riemannian geometry is now quantized: there are well-defined operators corresponding to, say, lengths, areas and volumes, and *all their eigenvalues are discrete.*

In quantum cosmology, one deals with symmetry reduced models. Since a reliable mathematical framework was not available in full quantum geometrodynamics, these models were treated ab-initio and quantized within the standard framework of quantum mechanics. By contrast, in loop quantum cosmology [5] one closely mimics the procedure used in full loop quantum gravity. The result turns out to be qualitatively different from the Wheeler DeWitt theory [4]. Specifically, because only the holonomy operators are well-defined and there is no operator corresponding to the connection itself, the von-Neumann uniqueness theorem is by-passed. A new representation of the algebra generated by holonomies and triads becomes available.[1] *We have new quantum mechanics* [9].

In loop quantum gravity, the standard Hamiltonian constraint is expressed in terms of the triads E and the curvature F of A. F can be expressed as a limit of the holonomy around a loop divided by the area enclosed by the loop, as the area shrinks to zero. Since there is no operator corresponding to A itself, in the quantum theory the limit does not exist. This is also a ramification of quantum geometry since area is quantized. Thus the quantum nature of geometry suggests that to obtain the quantum Hamiltonian constraint in quantum cosmology, one should shrink the loop only till it has the minimum non-zero eigenvalue of area. This is regarded as the fundamental operator; the expression in terms of the field strength emerges only in the classical limit where effects of quantum geometry can be neglected. As a result, in the resulting theory, the Wheeler-DeWitt differential equation is replaced by a *difference* equation (Eq (1) below), the size of the step being dictated by the first non-zero area eigenvalue —i.e., the 'area gap'— in quantum geometry. Qualitative differences from the Wheeler-DeWitt theory emerge precisely near the Big Bang singularity. Specifically, the evolution does not follow the classical trajectory. Quantum geometry gives rise to an effective negative pressure which becomes significant in the Planck regime. In effect, gravity becomes *repulsive* near the singularity and there is a quantum bounce.

[1] In the language of non-relativistic quantum mechanics, in the new representation there are 1-parameter unitary operators $\hat{U}(\lambda)$ 'quantizing' the classical functions $e^{i\lambda x}$. But they are not weakly continuous in the parameter λ whence there is no operator $\hat{x}$ corresponding to x itself [8]. In non-relativistic quantum mechanics, such representations are not of direct physical interest because we need position operator $\hat{x}$. In quantum cosmology, by contrast, full quantum geometry [1, 7] tells us that holonomies should be well defined but not connections themselves [9].

3 A Simple model

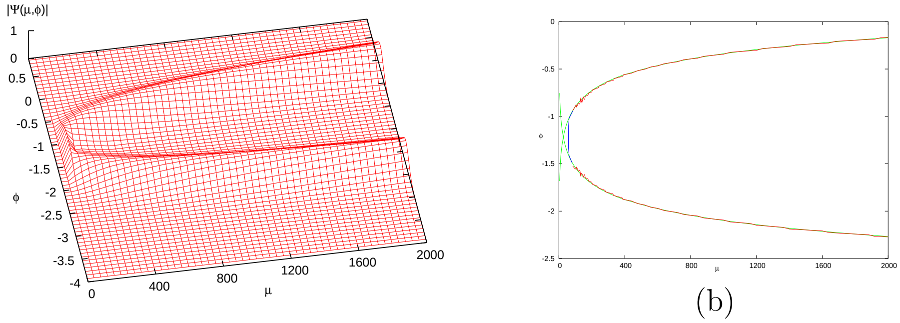
I will now illustrate these general features through a simple model: Homogeneous, isotropic $k = 0$ cosmologies with a zero rest mass scalar field [6]. The method also incorporates anisotropies, closed models and scalar fields with potentials. However it is more instructive to consider the simplest case because in this model the singularity is classically *inevitable*. One can make a plot of the scalar field versus the scale factor. The momentum p_ϕ of the scalar field is a constant of motion and for each value of p_ϕ, there are two trajectories, each with a singularity. In one, the universe starts out at the Big Bang and expands and in the other it contracts into a Big Crunch.

Classical dynamics suggests that here, as well as in the closed models, one can take the scalar field as an *internal clock* defined by the system itself —unrelated to any choice of coordinates or a background space-time. Can this idea be transported to quantum theory? The answer is in the affirmative. For, the Hamiltonian constraint equation

$$\frac{\partial^2 \Psi}{\partial \phi^2} = C^+(v)\Psi(v+4, \phi) + C^o(v)\Psi(v, \phi) + C^-(v)\Psi(v-4, \phi) \qquad (1)$$

can be regarded as 'evolving' the wave functions $\Psi(v, \phi)$ with respect to the 'internal time' ϕ. (Here v is the oriented volume (in Planck units) of a fiducial cell, so $v \sim \pm(\text{scale factor})^3$, and $C^\pm, C^o$ are simple algebraic functions on v.) The detailed theory is fully compatible with this interpretation. Thus, this simple model provides a concrete realization of the *emergent time* scenario, discussed in another session of this conference.

A standard ('group averaging') procedure enables one to introduce a natural Hilbert space structure on the space of solutions to the Hamiltonian constraint [6]. There are complete sets of Dirac observables using which one can rigorously construct semi-classical states and follow their evolution. Since we do not want to prejudice the issue by stating at the outset what the wave function should do at the singularity, let us specify the wave function at late time —say *now*— and, as the observational data demand, take it to be sharply peaked at a point on the expanding branch. Let us use the Hamiltonian constraint to evolve the state backwards towards the classical singularity. Computer simulations show that the state remains sharply peaked on the classical trajectory till very early times, when the density becomes comparable to the Planck density. The fluctuations are all under control and we can say that the continuum space-time of general relativity is an excellent approximation till this very early epoch. In particular, space-time can be taken to be classical at the onset of standard inflation. But in the Planck regime the fluctuations are significant and there is no unambiguous classical trajectory. This is to be expected. But then something unexpected happens. The state re-emerges on the other side again as a semi-classical state, now peaked on a contracting branch. Thus, in the Planck regime, although there are significant quantum fluctuations, the state has retained the 'memory' that



(b)

(a)

Figure 1: In these figures $\mu \sim$ scale factor^2 in Planck units. In Figure (a), a semi-classical wave packet is first constructed at a large value of μ, peaked at a point on a expanding classical trajectory with a given value of p_ϕ. It is then evolved 'backwards' in time using the Hamiltonian constraint of LQC. The absolute value of the wave function is plotted. Figure (b) shows the contracting and the expanding classical trajectories (for the same value of p_ϕ) *as well as* mean values of μ at each 'instant' ϕ, and fluctuations around these values, for the semi-classical state of figure (a). The quantum state remains peaked on the classical trajectory till densities become comparable to the Planck density. However, rather than continuing along this trajectory into the singularity as in the Wheeler-DeWitt theory, it then bounces and joins on to the second classical trajectory with the same value of p_ϕ.

it came from a semi-classical state. We do not have a quantum foam on the other side. Rather, there is a quantum bounce. Thus, *quantum geometry in the Planck regime serves as a bridge between two large classical universes.*

The fact that the state is again semi-classical in the past was unforeseen and emerged from detailed numerical simulations [6]. However, knowing that this occurs, one can develop an approximation scheme to derive an effective modification of the Friedmann equation that mimics the actual quantum evolution rather well:

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\,\rho\,[1 - \frac{\rho}{\rho_\star}] + \text{higher order terms} \tag{2}$$

where ρ is the matter density and $\rho_\star$, the critical density, is given by $\rho_\star = \text{const}\,(1/8\pi G\Delta)$, Δ being the smallest non-zero eigenvalue of the area operator. The key feature is that, without any extra input, the quantum geometry correction naturally comes with a *negative* sign making gravity repulsive in the Planck regime, giving rise to the bounce. The correction is completely negligible when the matter density is very small compared to the Planck density, i.e., when the universe is large. Finally, a key consequence of (1) is that the quantum evolution is deterministic across the 'quantum bridge'; no new input was required to 'join' the two branches. This is because, thanks to quantum geometry, one can treat the Planck regime fully non-perturbatively, without any need of a classical background geometry.

The singularity resolution feature is robust for the mini and midi-superspace models we have studied so far *provided* we use background independent description and quantum geometry. For example, in the anisotropic case, the evolution is again non-singular if we treat the full model non-perturbatively, using quantum geometry. But if one treats anisotropies as perturbations using the standard, Wheeler-DeWitt type Hilbert spaces, the perturbations blow up and the singularity is not resolved.

Finally, the Schwarzschild singularity has also been resolved. This resolution suggests a paradigm for the black hole evaporation process which can explain why there is no information loss in the setting of the physical, Lorentzian space-times [10]. The flaw in the semi-classical arguments is that they implicitly assume that the standard Penrose diagram of an evaporating black hole is valid everywhere, except possibly at the end point of the evaporation process. However, the diagram is a poor representation of the real physical situation *all along the singularity*, not just at the endpoint of the process. Quantum geometry resolves the singularity and the physical, quantum space-time is larger than what semi-classical considerations had us believe.

To summarize, quantum geometry effects have led to a resolution of a number of space-like singularities showing that quantum space-times can be significantly larger than their classical counterparts. These results have direct physical and conceptual ramifications.

I should emphasize however that so far the work has been restricted to mini and midi superspaces and a systematic analysis of generic singularities of the full theory is still to be undertaken. In the classical theory, singularities first appeared in highly symmetric situations. For a number of years, arguments were advanced that this is an artifact of symmetry reduction and generic, non-symmetric solutions will be qualitatively different. However, singularity theorems by Penrose, Hawking, Geroch and others showed that this is not correct. An astute use of the *geometric* Raychaudhuri equation revealed that singularities first discovered in the simple, symmetric solutions are in fact a generic feature. An interesting question is whether history will repeat itself and enable one to show that the singularity resolution resulting from *quantum geometry* is also rather generic.

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