Can we test effective quantum gravity with gravitational waves?

Nico Yunes

Institute for Gravitation and the Cosmos
Physics Department
The Pennsylvania State University

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S. Alexander and N. Y. “Parameterized post-Newtonian expansion of Chern-Simons gravity”, PRD75, 124022
S. Alexander, S. Finn and N. Y. “An observational probe of effective quantum gravity”, to be submitted soon.
A little motivation

- We would like a consistent unified quantum theory of Nature.
- Symmetry principles are useful to construct gauge theories.
- SM has CP violating interactions (eg, QCD sector), why not in gravity?
- In the absence of full quantum gravity, consider an “effective” theory:

  Effective Theory = GR + “CP-violation”

- Once we have formulated such a theory, can we test it?
  - Solar system effects? Gyroscopic precession. PPN tests.
The Chern-Simons Extension

- In analogy with QCD and inspired by CP violation, let us deform GR:

\[
S = \frac{1}{16\pi G} \int d^4x \left( \sqrt{-g} R + \frac{1}{4} f[\phi] \ast RR \right),
\]

where \( \ast RR = \epsilon^{\mu\nu\alpha\beta} R^\sigma_{\tau\alpha\beta} R^\tau_{\sigma\mu\nu} / 2 \) and \( f[\phi] \) is some functional of some “external” dynamical field \( \phi \) (Jackiw & Pi).

- Varying the action wrt \( g_{\mu\nu} \): \( \delta (f \ast RR) \to \sqrt{-g} C_{\mu\nu} \delta g^{\mu\nu} \) and the EEs become:

\[
G_{\mu\nu} + C_{\mu\nu} = 8\pi T_{\mu\nu}
\]

where \( C_{\mu\nu} \) is the Cotton tensor given by

\[
C_{\mu\nu} \propto \left[ f;_{\sigma} \epsilon^{\sigma\alpha\beta}_{(\mu R_{\nu})_{\beta;\alpha}} + f;_{\tau} \ast R^\tau_{(\mu} \sigma_{\nu)} \right]
\]

- Like in QCD, we have now a \textbf{P violating effective theory} with a new scalar field \( \phi \) (the gravitational axion).
Effective Quantum Gravity
Who ordered that?

Quantum theory suggests such a Chern-Simmons extension

- **String Theory:**
  - A Chern-Simmons-like term is needed to cancel a chiral anomaly.
  - Otherwise, string theory would not be a mathematically consistent QFT.
  - If quantum origin, then likely to be suppressed by Planck scale.
  - But some scenarios postulate enhancement of $f$ via coupling to curvature and mass current.

- **Loop Quantum Gravity:**
  - No effective Hamiltonian in the full theory, yet. However, if ST needs it . . .

But the correction need not be quantum inspired!

- There other scenarios where the Chern-Simons correction arises from topological considerations.
Solar System Tests

The ABC of PPN

A) Solve the EOM in your alternative theory.
   - Expand the modified Einstein Equations about a Minkowski background to second order in the metric perturbation (Nordtvedt & Will).
   - Assume a perfect fluid stress-energy source (binaries, Earth-Sun, etc).
   - Assume a slow-motion/weak-gravity approximation and perturbatively solve the linearized EEs: $g_{00}$ to $\mathcal{O}(v^4)$, $g_{0i}$ to $\mathcal{O}(v^3)$ and $g_{ij}$ to $\mathcal{O}(v^2)$.
   - The final solution is then expressed in terms of PPN potentials (Green-function like integrals over the stress energy tensor.)

B) Construct a Super-Duper-Duper Metric
   - Based on some assumptions, construct a family of metric solutions with PPN potentials, with the family labeled by some PPN parameters, eg.

   $$g_{ij} = (1 + 2\gamma U) \delta_{ij}$$

C) Compare solution to new theory to super-metric and read off PPN params.

Has that parameter been measured? → You’ve tested your theory!
Can we test CS in the Solar System?
(Alexander & Yunes)

- When we expand the Cotton tensor in the PPN framework, we find that $C_{00} = \mathcal{O}(v)^6$, $C_{ij}$ leads to $\delta h_{ij} = \mathcal{O}(\dot{f})^2$ and

$$C_{0i} \sim -\frac{1}{4} \dot{f} \varepsilon^{kl} i \nabla^2 h_{0l,k}$$

- The only modification to the gravitational field to leading $\mathcal{O}(\dot{f})$ is then

$$\delta h_{0i} \sim 2 \dot{f} (\nabla \times V)_i$$

where $V_i$ is one of the PPN potentials (lowest-order vector potential).

- New PPN parameter!! (such terms had not been considered before.)

- Specialize to a binary sys. of pt. ptcls. : $V^i \sim m_A v_A^i / r_A - (n_A \times J_A)^i / (2 r_A^2)$

$$g^{(1)}_{0i} = -\frac{7}{2} \frac{m_1}{r_1} v_1^i - \frac{m_1}{6 r_1^2} \left( v_1 - v_1^{(eff)} \right) - \frac{1}{2} n_1 \frac{m_1}{r_1} \left( v_1^{(eff)} \cdot n_1 \right) - 2 \left( \frac{J_1^{(eff)}}{r_1^2} \times n_1 \right)$$

where $v_{A,\text{eff}} = v_A^i - 6 \dot{f} J_A^i / (m_A r_A^2)$ and $J_A^{i(\text{eff})} = J_A^i - \dot{f} m_A v_A^i$

- Axion is a fluid that is “dragged” by motion and couples to ang. mom. $\sim$ Kerr.
Acceleration and Frame-Dragging

- Gravitomagnetic analogy: Construct electromagnetic vectors via

\[ E^i = - (\nabla \Phi)^i - \dot{A}^i / 2 \quad \text{and} \quad B^i = (\nabla \times A)^i, \]

where \( A^i \propto g_0 i \). Thus, CS modifies the gravitomagnetic sector of \( ds^2 \).

- Gravitomagnetism affects the acceleration of point particles, namely

\[ \delta a^i = \frac{1}{8} \delta g^i + \frac{1}{2} (v \times \delta \Omega)^i = -\frac{3}{2} \dot{f} (v_1 \cdot n_1) (v_1 \times n_1)^i + 1 \rightarrow 2, \]

but for circular orbits the CS correction vanishes (\( v \perp n \)).

- However, CS also affects directly the frame-dragging of gyroscopes, where

\[ \delta \Omega^i = - \sum_A \dot{f} \frac{m_A}{r_A^3} \left[ 3 (v_A \cdot n_A) n_A^i - v_A^i \right]. \]

- Detectable? Smith, Erickcek, Caldwell and Kamionkowski recently expanded our analysis to account for extended objects and found that \( \dot{f} < 10^{-2} \) seconds using LAGEOS.
Gravitational Waves and Birefringence

- Expand the EEs about a FRW background since redshift might enhance effect by accumulation (Alexander & Martin)

$$\Box_g h_L = -i F[f, \ddot{f}] \dot{h}_{L,i} \hat{k}^i \quad \Box_g h_R = +i F[f, \ddot{f}] \dot{h}_{R,i} \hat{k}^i$$

with solution $h_{R,L} = e^{\pm f\tau} h_{R,L}^0 \rightarrow$ the CS correction introduces an exponential growth/decay of the L/R polarization ("movie").

- Pick a binary and then (Alexander, Finn & Yunes):

$$h_+ = \frac{2M}{d_L} (\pi M f)^{2/3} \left[ (1 + c_i^2) C_\phi \cosh (f\tau) - 2c_i S_\phi \sinh (f\tau) \right]$$

$$h_\times = \frac{4M}{d_L} (\pi M f)^{2/3} \left[ c_i S_\phi \cosh (f\tau) + \frac{1}{2} (1 + c_i^2) C_\phi \sinh (f\tau) \right]$$

where $M$ is the chirp mass, $f$ is the GW frequency, $d_L$ is the luminosity distance $c_i = \cos i$ with $i$ the inclination angle and $\phi$ the GW phase.

- Can we test CS correction with LISA? Fisher analysis says YES!: $\dot{f} \lesssim 10^{-3}$ secs. (rough estimate, making several assumptions on source and instrument).
Conclusions

• Inspired by CP violation in QCD, one can construct an effective CP-violating gravity theory: Chern-Simons gravity.

• Solar system effects (frame-dragging), studied via weak-field and PPN analysis. Extending Alexander & Yunes, Smith, et al. place bound with LAGEOS at the 1% level.

• Gravitational wave effects difference in polarization amplitude might be detectable by LISA if $\delta \phi$ is large enough. Possible bounds might be one or two orders of magnitude larger than Solar system ones.

Future Work

• How large can $f$ be? Possible enhancements via coupling with neutron currents and regions of high curvature.

• Can we do better with LISA by performing a more detailed Fisher analysis?

• Are there other Solar System effects?
...and, obviously, $G_{ab} = T_{ab}$...

...you can now ask Nico some questions...

THANKS